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Van der Pol Approximation Applied to Wien Oscillators

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Abstract

This paper presents a nonlinear analysis of the Wien type oscillators based on the Van der Pol approximation. The steady-state equations for the key parameters, frequency and amplitude, are derived and their sensitivities to ambient temperature are discussed. The added value of this work is to present an analytical method to obtain the fundamental characteristics of the Wien type oscillators and to relate these characteristics with the circuit parameters in a simpler manner. The simulation results confirm the amplitude and frequency equations, and confirm that the amplitude is controlled by the limiter circuit.

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Keywords:
Oscillator, Wien oscillator, quasi-sinusoidal, Van der Pol, amplitude stabilization.

1. Introduction

The oscillators of the Wien type are well known, second order, quasi-sinusoidal oscillators, suited for low frequencies and low distortion applications [1]. The low-distortion feature of this type of oscillator justifies its use for the characterization of analog-to-digital converters (ADC)[2], instrumentation circuits and a wide variety of measurement circuits. More recently, the interest for this type of oscillator has been increasing because of the demand for full integration of complete systems in a single chip, eliminating the off-chip components [3]. For these reasons, the oscillator of the Wien type is a current topic. There is extensive literature on linear analysis, but neglects important performance parameters, e.g. harmonic distortion, and cannot be used to determine the oscillation amplitude. Also, there is literature and several works addressing the nonlinear analysis using the phasor notation and the Van der Pol approximation [1,4], presenting optimizations to lower harmonic distortion. Although the oscillation amplitude and the circuit parameters are related, a simple and comprehensive equation is missing, or uses fitting parameters, in particularly for temperature variation. The aim of this work is to obtain simpler equations for the oscillation amplitude. First, we analyze the amplitude limiter based on an incandescent light bulb (lamp) and then the anti-parallel diodes limiter. For the former the analysis is based on the laws of thermodynamics [5] and the latter is based on the Shockley's equation [6].

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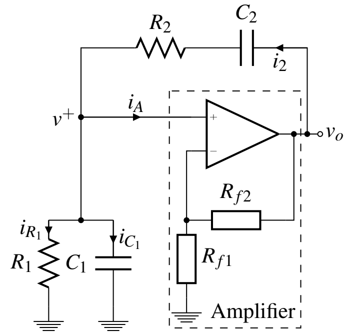


Fig. 1: Generic Wien-Oscillator

In the next section we present the generic oscillator of the Wien type and derive its fundamental equations. In Section 3, we present the analysis of the amplitude stabilization circuits using the Van der Pol approximation. We analyze amplitude limiters circuits with a lamp and with diodes. In Section 4, we provide a discussion of the analytical and simulations results. In Section 5 the conclusions are drawn.

2. Wien type Oscillator

The generic circuit of the Wien-oscillator is shown in Fig. 1. As will be seen in the next section, the amplitude limiter is implemented by using nonlinear elements in the amplifier feedback network. Therefore, we will derive the set of fundamental equations in the time domain. Applying the Kirchhoff's current law (KCL) at node v^+ results

$$i_2 = i_{R_1} + i_{C_1} + i_A, \tag{1}$$

where i_{R_1} is the current passing through resistor R_1, i_{C_1} is the current passing through capacitance C_1 and i_A is the input current of the operational amplifier (ampop), which can be neglected if we assume that the ampop is ideal.

The current passing through R_1 is

$$i_{R_1} = \frac{v^+}{R_1} = \frac{1}{R_1 G} v_o, \tag{2}$$

where G is the amplifier gain. The current passing through C_1 is

$$i_{C_1} = \frac{C_1}{G}\frac{\mathrm{d}v_o}{\mathrm{d}t} - \frac{C_1}{G^2}\frac{\mathrm{d}G}{\mathrm{d}t} v_o. \tag{3}$$

Notice that in (3) we include the derivative of the gain because of its nonlinear nature and its dependence on the output voltage. Substituting (2) and (3) into (1) results

$$i_2 = \frac{1}{R_1 G} v_O + \frac{C_1}{G}\frac{\mathrm{d}v_O}{\mathrm{d}t} - \frac{C_1}{G^2}\frac{\mathrm{d}G}{\mathrm{d}t} v_O. \tag{4}$$

The output voltage is

$$v_o\left(1 - G^{-1}\right) = R_2 i_2 + \frac{1}{C_2}\int i_2 \mathrm{d}t, \tag{5}$$

substituting (4) into (5) and rearranging the terms one obtains

$$\frac{\mathrm{d}^2 v_o}{\mathrm{d}t^2} + \left[\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1-G}{R_2 C_1} - \frac{1}{G}\frac{\mathrm{d}G}{\mathrm{d}t} + \right]\frac{\mathrm{d}v_o}{\mathrm{d}t} + \left[\frac{1}{R_1 C_1 R_2 C_2} - \frac{1}{R_2 C_1}\frac{\mathrm{d}G}{\mathrm{d}t} + \frac{1}{G^2}\left(\frac{\mathrm{d}G}{\mathrm{d}t}\right)^2 - \frac{1}{G}\frac{\mathrm{d}^2 G}{\mathrm{d}t^2}\right] v_o = 0. \tag{6}$$

Assuming no mismatches, i.e. $R = R_1 = R_2$ and $C = C_1 = C_2$, and a small variation of the gain, $\frac{dG}{dt} \ll 1$, one obtains the second-order differential equation

$$\frac{d^2 v_o}{dt^2} + \frac{1}{RC}(3-G)\frac{dv_o}{dt} + \left(\frac{1}{RC}\right)^2 v_o \approx 0. \tag{7}$$

Assuming constant coefficients, i.e. components fixed values, the solution, derived in [7], is of the form

$$v_o(t) = A_m(t)\sin(\omega_o t + \phi), \tag{8}$$

where A_m is the amplitude of oscillation, ω_0 the oscillation frequency and ϕ is the phase. From the middle term of (7) (damping term), it is clear that for the oscillation to start the damping term must be negative, i.e. $(3-G) < 0$, which leads to the limit condition of $G > 3$. Since the gain is given by

$$G = \frac{R_{f2}}{R_{f1}} + 1, \tag{9}$$

means that, the condition $\frac{R_{f2}}{R_{f1}} > 2$ has to be met to start the oscillation. The oscillation frequency is

$$\omega_0 = \frac{1}{RC}. \tag{10}$$

The phase, ϕ, can only be determined by the circuit initial conditions. For the amplitude A_m, the only conclusion that can be drawn, from (7), is the unlimited increase for $G > 3$. However, in a practical implementation, the amplitude will be limited by the ampop maximum output voltage, which is an undesirable limitation, because it distorts the output signal. To minimize distortion, designers usually use amplitude control circuit, as shown in the following section.

3. Amplitude Control

The automatic amplitude control is done by the adjustment of the amplifier gain. For small voltage amplitudes the ratio $\frac{R_{f_2}}{R_{f_1}} > 2$ leads to an exponential grow of the amplitude. When the amplitude approaches the steady-state value, the gain is proportionally reduced, to $\frac{R_{f_2}}{R_{f_1}} = 2$, and the amplitude stabilizes. As will be shown in the next subsections the automatic gain control can be obtained with a nonlinear element in R_{f_1}, or R_{f_2} [6,8–10].

3.1. Tungsten lamp

The amplitude limiter using an incandescent lamp was first proposed by Meacham in [11], and improved in [12,13]. The modern version of the oscillator is shown in Fig. 2a. The operation principle is based on thermal feedback, meaning that the amplitude depends on the filament temperature, and the temperature depends on the dissipated power (joule effect). Hence, the resistance increases when the output voltage rises leading to a reduction of the gain.

The electrical resistance of the lamp, like that of most metals [5], can be approximated, for a limited temperature range, by

$$R_{f1} \approx R_0\left[1 + \alpha(T - T_0)\right], \tag{11}$$

where R_0 is the resistance at the reference temperature (T_0), α is the temperature coefficient of the resistance and T is the filament temperature. To determine the relation between the resistance and the amplitude, it is necessary to determine first the relation between the dissipated power and the filament temperature, see the model of the lamp in Fig. 2b. The heat accumulated rate, in the filament, is equal to the applied electrical power minus the heat loss rate[5,14], given by

$$C_T \frac{dT_\Delta}{dt} = P(t) - \frac{1}{R_T} T_\Delta, \tag{12}$$

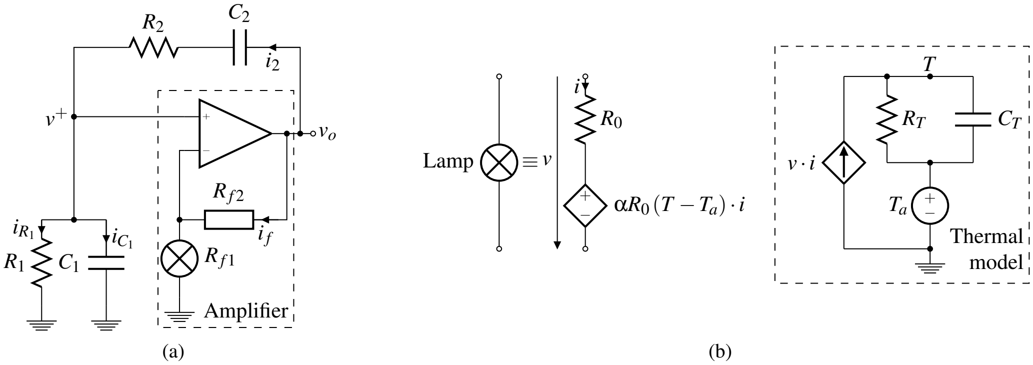


Fig. 2: a) Wien-Oscillator with lamp based amplitude control, b) lamp's model

where P is the power dissipated in the lamp, R_T is the thermal resistance of the lamp (dissipation constant), C_T is the thermal capacitance of the lamp and T_Δ is the difference between ambient and filament temperature.

The power dissipated at steady state, where $R_{f_2} \approx 2R_{f_1}$, can be approximated by

$$P(t) \approx R_{f_1} \frac{v_o^2}{\left(\frac{3}{2} R_{f_2}\right)^2}. \tag{13}$$

Substituting (8) into (13) yields

$$P(t) \approx \frac{R_{f_1}}{\left(\frac{3}{2} R_{f_2}\right)^2} \frac{A^2}{2} \left[1 + \cos\left(2\omega_0 t\right)\right] = P_0 \left[1 + \cos\left(2\omega_0 t\right)\right], \tag{14}$$

where P_0 is the average power dissipated. Substituting (14) into (12) one obtains the differential equation which describes the filament temperature

$$\frac{\mathrm{d}T_\Delta}{\mathrm{d}t} + \omega_T T_\Delta \approx \frac{1}{C_T} P_0 \left[1 + \cos\left(2\omega_0 t\right)\right], \tag{15}$$

with the solution

$$T_\Delta \approx \frac{1}{\omega_T C_T} P_0 \left[\left(1 - e^{-\omega_T t}\right) + \underbrace{\frac{1}{\sqrt{1 + \left(\frac{2\omega_0}{\omega_T}\right)^2}}}_{k(\omega_0, \omega_T)} \left[\cos(2\omega_0 t + \phi) - e^{-\omega_T t} cos(\phi)\right] \right]. \tag{16}$$

For steady state, i.e. $t \to \infty$, (16) is reduced to

$$T \approx T_a + \frac{1}{\omega_T C_T} P_0 \left[1 + k(\omega_0, \omega_T) \cos(2\omega_0 t + \phi)\right]. \tag{17}$$

It can be seen from (17) that the filament temperature depends: on the ambient temperature; on the average power, P_0, and on the second-harmonic power. The lamp behaves as a low-pass filter, attenuating the second harmonic by k.

Substituting (17) into (11) and assuming that $T_0 = T_a$ results

$$R_{f_1} = R_0 \left(1 + \alpha \frac{1}{\omega_T C_T} P_0 \left[1 + k(\omega_0, \omega_T) \cos(2\omega_0 t + \phi)\right]\right). \tag{18}$$

Substituting (14) into (18) and solving for R_{f_1} gives

$$R_{f_1} = \frac{R_0}{1 - \frac{4\alpha R_T R_0}{9R_{f_2}^2}\frac{A^2}{2}\left(1 + k(\omega_0, \omega_T)\cos(2\omega_0 t + \phi)\right)}. \tag{19}$$

Using (19) into (9) and in (7) one obtains the gain

$$G = 1 + \frac{R_{f_2}}{R_0} - \frac{4\alpha R_T}{9R_{f_2}}\frac{A^2}{2}\left[1 + k(\omega_0, \omega_T)\cos(2\omega_0 t + \phi)\right]. \tag{20}$$

Substituting (20) into (7) and after extensive calculations, using the harmonic balance method, (7) can be rewritten in the general form of the Van der Pol oscillator [7],

$$\frac{\mathrm{d}^2 v_o}{\mathrm{d}t^2} - 2\left(\delta_0 - \delta_2 v_o^2\right)\frac{\mathrm{d}v_o}{\mathrm{d}t} + \omega_0^2 v_o = 0, \tag{21}$$

where

$$\delta_0 = \frac{\omega_0}{2}\left(\frac{R_{f_2}}{R_0} - 2\right), \tag{22}$$

and

$$\delta_2 = \frac{\omega_0}{2}\frac{8\alpha R_T}{9R_{f_2}}\left(1 + k/2\right). \tag{23}$$

The steady-state amplitude of the Van der Pol oscillator is given by

$$A_m = 2\sqrt{\frac{\delta_0}{\delta_2}}, \tag{24}$$

substituting (22) and (23) into (24) one obtains the oscillation amplitude for the circuit of Fig. 2a,

$$A_m = 3\sqrt{\frac{1}{2\alpha R_T}}\sqrt{\frac{\left(\frac{R_{f_2}}{R_0} - 2\right)R_{f_2}}{1 + k/2}}. \tag{25}$$

Notice that only for $\frac{R_{f_2}}{R_0} > 2$ makes (25) valid, which is the limit condition for the start of oscillation. Also notice that R_0 is temperature dependent, meaning that the amplitude must be fitted if the ambient and reference temperatures, T_a and T_0, differ.

3.2. Diodes in anti-parallel configuration

The use of diodes in the amplifier feedback network limits the oscillation amplitude [6], as shown in Fig. 3a.

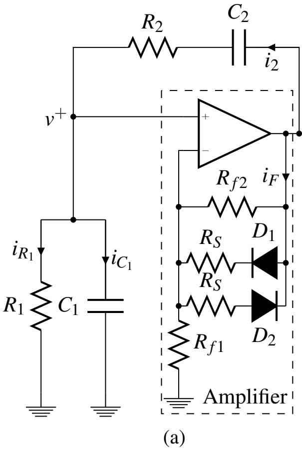
The diode current is given by the Shockley equation

$$i_D = I_s\left(e^{\frac{v_D}{nV_T}} - 1\right), \tag{26}$$

where I_s is the reverse bias saturation current, v_D is the voltage across the diode, n is the emission coefficient and V_T is the thermal voltage, see [15]. The feedback current is

$$i_F \approx= \frac{V_D}{R_{f_2}} + I_s\left(e^{\frac{v_D}{nV_T}} - e^{\frac{-v_D}{nV_T}}\right), \tag{27}$$

and the Taylor series approximation gives



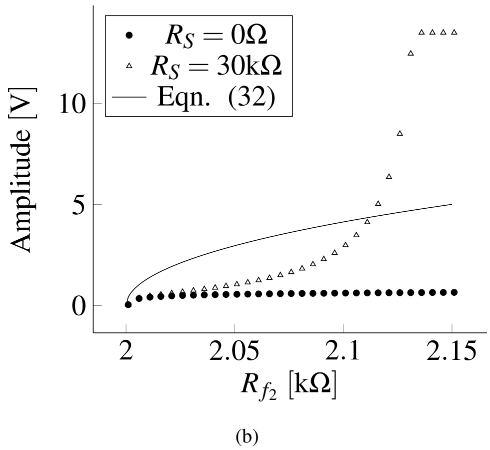


Fig. 3: a) Wien-Oscillator with anti-parallel diodes, b) Simulation results

$$i_F \approx \frac{v_D}{R_{f_2}} + 2I_s \left[\frac{v_D}{nV_T} + \left(\frac{v_D}{3!nV_T} \right)^3 + \dots \right], \tag{28}$$

where $v_D = (v_o - v^-) - R_S i_D$ is the voltage at the diode terminals and the dots represent the high order terms. Assuming that $R_S = 0\Omega$, $\frac{2I_s R_{f_1}}{nV_T} \ll 1$ and an ideal ampop, the output voltage is given by

$$v_O \approx \underbrace{\left(\frac{R_{f_2}}{R_{f_1}} + 1 \right)}_{G_0} v^+ - \frac{I_s R_{f_2}}{3(nV_T)^3} (G_0 - 1)^3 \, {v^+}^3. \tag{29}$$

From (29) it is possible to obtain, for steady-state where $G \approx 3$, the amplifier gain

$$G \approx G_0 - \frac{8 I_s R_{f_2}}{9 (nV_T)^3} v_o^2. \tag{30}$$

Substituting (30) into (7) yields

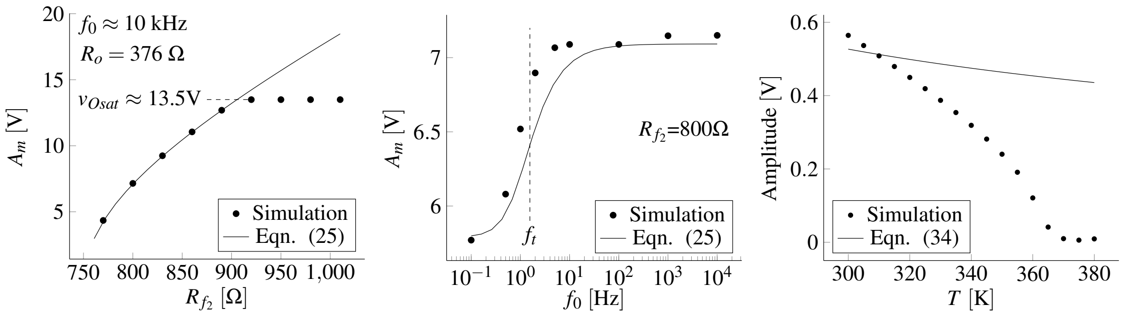
$$\frac{\mathrm{d}^2 v_o}{\mathrm{d}t^2} + \omega_0 \left[(3 - G_0) + \frac{8 I_s R_{f_2}}{9 (nV_T)^3} v_o^2 \right] \frac{\mathrm{d}v_o}{\mathrm{d}t} + \omega_0^2 v_o = 0, \tag{31}$$

which can be rewritten in the Van der Pol form, see (21), where $\delta_0 = \frac{\omega_0}{2}(G_0 - 3)$ and $\delta_2 = \frac{\omega_0}{2} \frac{8 I_s R_{f_2}}{9(nV_T)^3}$. Hence, the amplitude is

$$A_m = 2\sqrt{\frac{\delta_0}{\delta_2}} = 3\sqrt{\frac{n^3 V_T^3 \left(\frac{R_{f_2}}{R_{f_1}} - 2 \right)}{2 R_{f_2} I_s}}. \tag{32}$$

However, from (32), the temperature dependence is not clear since I_S and V_T are both dependent on the junction temperature. Considering the junction temperature effect on the saturation current I_S [15,16] given by

$$I_S = I_{S0} \left(\frac{T}{T_0} \right)^\gamma e^{\frac{E_g}{kT} \left(\frac{T}{T_0} - 1 \right)}, \tag{33}$$



(a) (b) (c)

Fig. 4: Amplitude of oscillation

Table 1 Amplitude error

	Capacitor C [nF]								
R_{f_2} [Ω]	1	10	100	1000	2000	5000	10000	20000	100000
770	-1.91%	-2.44%	-0.05%	-1.65%	-3.03%	-5.04%	-4.60%	-2.15%	0.24%
800	-0.81%	-0.79%	-0.15%	-1.84%	-3.30%	-5.40%	-4.85%	-2.14%	0.49%
830	-0.39%	-0.38%	-0.18%	-1.95%	-3.48%	-5.67%	-5.04%	-2.08%	0.77%
860	-0.19%	-0.25%	-0.20%	-2.04%	-3.63%	-5.91%	-5.22%	-2.03%	1.03%
890	-0.05%	-0.11%	-0.22%	-2.13%	-3.78%	-6.14%	-5.39%	-1.98%	1.26%

where I_{S0} is the saturation current at the reference temperature (T_0), E_g is the energy gap constant, γ is the temperature exponent term, k is the Boltzmann constant and T is the absolute temperature of the p-n junction. Substituting (33) into (32) and assuming $\gamma = 3$ we obtain

$$A_m \approx 3\sqrt{\frac{n^3 V_{T_0}^3 T_0}{2R_{f_2} I_{S0} T}}\sqrt{\left(\frac{R_{f_2}}{R_{f_1}} - 2\right)}. \tag{34}$$

where V_{T_0} is the thermal voltage at the reference temperature (T_0) and T is the absolute temperature of the p-n junction. From (34) it is clear that the amplitude depends inversely of the p-n junction temperature, hence, the amplitude decreases when the temperature increases.

4. Simulation results

The circuits shown in Fig. 2a and Fig. 3a are simulated in PSpice version 16.3. The circuit parameters are C=1 nF, R=15.9 kΩ, $R_{f_2} = 800\Omega$ and the ampop TL081 model is used. The ampop is powered with ±15 V. The lamp parameters are R_T=913.71kΩ, $C_T = 111.7$ nF and $\alpha = 0.01$ Ω/K. For the anti-parallel diodes circuit, the model of the 1N4148 diode is used.

For the lamp limiter, an amplitude error below 2% is obtained in all simulations at frequencies far (at least one decade) from the lamp's thermal pole frequency, f_T. For oscillation frequencies near f_T the error increases to 6%, as can be seen in Fig. 4b and in Table 1. A possible explanation for the increase of the error, near f_T, is the disregard of the phase shift added by the lamp. Fig. 4b also shows that an higher oscillation amplitude is required for frequencies above f_T, as shown by (17), since the lamp behaves like a low-pass filter attenuating the average power of the second harmonic. Therefore, for frequencies above f_T more amplitude is needed to increase the value of R_{f_1} and reach the ratio, $R_{f_2}/R_{f_1} = 2$, that stabilizes the amplitude. The simulation result, in Fig. 4a, confirms the theory in (25). In the amplifier linear zone the error is below 2%, as can be seen in Table 1. The amplifier output voltage reaches saturation, at 13.5 V, for values of R_{f_2} above 900 Ω, which explains the deviation from the theoretical values.

For the limiter based on the anti-parallel diode, a comparison between the estimated amplitude and the simulation results is shown in Fig. 3b, where the theory and the simulation have a significant mismatch. This mismatch can be explained by the diodes' non linearity, meaning that the Van der Pol approximation (third order) is not sufficient, an order of seventh, or higher, is required. The temperature dependence shows the correct trend, however, the error between simulation and theory is significant, see Fig. 4c.

5. Discussion and Conclusions

A nonlinear analysis of the oscillators of Wien type for the amplitude-stabilizing mechanism has been presented. The fundamental characteristics of the oscillator were derived and their direct relations with circuit parameters were obtained. The results show that the oscillation frequency is independent of the amplitude control block, hence, only depends on the RC network, which is a well-known characteristic of this oscillator.

For the lamp-stabilized oscillator, the amplitude equation is validated by the simulation results, which achieved errors around 2% for oscillation frequencies far from lamp's thermal pole frequency. The lamp is a low-pass filter, which attenuates the second harmonic of the dissipated power, thus, for higher frequencies, a higher amplitude is required to increase the lamp resistance. We can conclude that to maximize the oscillation amplitude the designer should use a lamp with a pole frequency below the working frequency of the oscillator.

For the anti-parallel diodes oscillator, the amplitude is very sensitive to the diode's saturation current, which makes the amplitude estimation highly inaccurate. Further adjustments to the approximation are needed. Such as, increasing the order of the polynomial approximation. Nevertheless, the amplitude dependence on the ambient temperature is important, since it reduces the oscillation amplitude and for higher temperatures can stop the oscillation. Therefore, the designer should not disregard the influence of the ambient temperature.

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